Model-independent analysis of lepton flavour violating τ decays

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ABSTRACT: Many models for physics beyond the Standard Model predict lepton-flavour violating decays of charged leptons at a level which may become observable very soon. In the present paper we investigate the decays of a τ into three charged leptons in a generic way, based on effective-field-theory methods, where the relevant operators are classified according to their chirality structure. We work out the decay distributions and discuss phenomenological implications.

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Contents

1. Introduction

Lepton flavour violation (LFV) has become a hot topic over the last few years. On the one hand, the discovery of neutrino oscillations in combination with the minimal extension of the Standard Model (SM) predicts lepton flavour violation for the charged leptons, however, at a completely unobservable level. On the other hand, many extensions of the SM predict LFV at much higher rates, which, in some cases, may already be in conflict with existing experimental bounds [1] (see also [2] for a recent summary of B-factory results). With the advent of new experimental facilities [3] (see also [4]) the current bounds will be pushed further, if not a discovery will be made. In particular, at the LHC experiments it will be possible to detect LFV decays of a τ lepton, especially into channels with three leptons; here the signal $\tau \to 3\mu$ will be one of the cleanest signatures [5].

There are many models which predict LFV τ decays of the form $\tau \to \ell\ell'\ell''$ with $\ell, \ell', \ell'' = e, \mu$ [6 – 17]. All these models will eventually match onto a set of local four-fermion operators or radiative operators, the latter mediating $\tau \to \ell\gamma^*$ with subsequent decay of the (virtual) photon into a charged lepton pair. In a bottom-up approach, this allows us to consider all possible four-fermion and radiative operators with arbitrary coupling constants, which can be determined by studying the decay distributions of the three leptons in the final state. Even if no signal events are found, such a study of the decay distributions is necessary to determine the efficiency of an experiment and hence to extract reliable limits.

Our paper is organized as follows. In the next section we will introduce the general set of effective operators of dimension six and eight, which mediate the decays $\tau \to \ell\ell'\ell''$ at the electroweak scale. This includes four-fermion operators with scalar, vector and tensor currents as well as radiative operators contributing to $\tau \to \ell\gamma \to \ell\ell'\ell''$. These operators match onto the relevant low-energy interactions at the scale of the τ mass, which are parameterized by a number of unknown coupling constants. We will also give

a brief discussion on how the couplings are related to the lepton masses and the PMNS neutrino mixing matrix within minimal-flavour violating scenarios [18]. Focusing on the decays $\tau^- \to \mu^- \mu^- \mu^+$ in section 3.1 and $\tau^- \to e^- \mu^- \mu^+$ in section 3.2, we calculate the Dalitz distributions for the individual chirality structures appearing in the effective Hamiltonian, taking into account interference terms apart from corrections of order m_μ/m_τ. We conclude with a brief comparison of existing results on LFV processes in specific new physics scenarios in section 4.

2. The effective interaction for $\tau \to \ell\ell'\ell''$ decays

In this paper we follow a bottom-up approach to lepton-flavour violating decays, using an effective-field-theory picture. We assume that some new physics at a high scale Λ induces lepton-flavour violating processes.[1] At the electro-weak scale these lepton-flavour-violating interactions manifest themselves in higher dimensional operators which have to be compatible with the $SU(2)_L \times U(1)_Y$ gauge symmetry of the SM. The leading operators for processes involving charged leptons will be of dim=6 and dim=8, which we shall construct in the following (similar analyses within the context of supersymmetric extensions of the SM can be found, for instance, in [8, 10]).

To this end we group the left-handed leptons in an $SU(2)_L$ doublet, while the right-handed charged leptons (which are singlets under $SU(2)_L$) are put into an incomplete doublet, as a reminiscent of a right-handed $SU(2)_R$ related to custodial symmetry. Writing also the Higgs boson in matrix form, we have

$$L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}, \quad R = \begin{pmatrix} 0 \\ \ell_R \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h_0 + i\chi_0 & \sqrt{2}\phi_+ \\ -\sqrt{2}\phi_- & v + h_0 - i\chi_0 \end{pmatrix}. \quad (2.1)$$

For simplicity, we have suppressed the family indices, which will be specified once we consider a particular decay mode. For later use, we note that the hypercharge Y is defined in terms of the right-handed generator $T_3^{(R)}$ as

$$Y = T_3^{(R)} + \frac{1}{2}(B - L). \quad (2.2)$$

In terms of the fields defined in (2.1), the list of operators relevant for $\tau \to \ell\ell'\ell''$ decays reads

dim=6 leptonic:

$$O_1 = (\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L) \quad (2.3)$$

$$O_2 = (\bar{L}\tau^a \gamma_\mu L)(\bar{L}\tau^a \gamma^\mu L) \quad (2.4)$$

$$O_3 = (\bar{R}\gamma_\mu R)(\bar{R}\gamma^\mu R) \quad (2.5)$$

$$O_4 = (\bar{R}\gamma_\mu R)(\bar{L}\gamma^\mu L) \quad (2.6)$$

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[1]Notice that, in general, the scale associated to lepton-flavour violation is independent of the scale related to lepton-number violation, Λ_{LN}.

dim=6 radiative:

$$R_1 = g'(\bar{L}H\sigma_{\mu\nu}R)B^{\mu\nu} \tag{2.7}$$

$$R_2 = g(\bar{L}\tau^a H\sigma_{\mu\nu}R)W^{\mu\nu,a} \tag{2.8}$$

dim=8 leptonic:

$$P_1 = (\bar{L}HR)(\bar{L}HR) \tag{2.9}$$

$$P_2 = (\bar{L}\tau^a HR)(\bar{L}\tau^a HR) \tag{2.10}$$

$$Q_1 = (\bar{L}HR)(\bar{R}H^\dagger L) \tag{2.11}$$

$$Q_2 = (\bar{L}\tau^a HR)(\bar{R}H^\dagger \tau^a L) \tag{2.12}$$

$$P_1^{(T)} = (\bar{L}H\sigma_{\mu\nu}R)(\bar{L}H\sigma^{\mu\nu}R) \tag{2.13}$$

$$P_2^{(T)} = (\bar{L}\tau^a H\sigma_{\mu\nu}R)(\bar{L}\tau^a H\sigma^{\mu\nu}R) \tag{2.14}$$

where $B_{\mu\nu}$ is the $U(1)_Y$ gauge field, $W_{\mu\nu}^a$ are the $SU(2)_L$ gauge fields, and g and g' are the corresponding gauge couplings. In the above list we have only shown operators that have tree-level contributions to leptonic τ-decays; more operators, which are bi-linear in the lepton fields and contribute at the loop level, can be found e.g. in [8, 10, 18]. We also neglected dim=8 operators involving additional covariant derivatives. When acting on fermions, the derivatives become fermion masses by the equations of motions, such that these operators are additionally suppressed by the small lepton Yukawa couplings. Notice that the leptonic dim=6 operators $O_1 - O_4$ only contain helicity conserving currents. The most general effective Hamiltonian at the electro-weak scale is then obtained by summing over these operators, multiplied by arbitrary coefficients for every flavour combination. In a particular new physics scenario, these coefficient should be obtained by matching at the new physics scale Λ and evolving down to the scale M_W within the SM (as an effective theory).

In the following we are interested in LFV decays of a τ lepton into three charged leptons. To this end we have to construct the effective interaction at the scale of the τ lepton, by integrating out the weak gauge bosons and the Higgs. We will focus on τ^- decays; the decay distributions for τ^+ decays are identical. On the level of four-fermion operators with dim=6, we obtain the same structures as in (2.3)–(2.6). Projecting on charged leptons only, we see that O_2 becomes equivalent to O_1, and both match onto a purely left-handed operator

$$H_{\text{eff}}^{(LL)(LL)} = g_V^{(LL)(LL)} \frac{(\bar{\ell}_L\gamma_\mu\tau_L)(\bar{\ell}'_L\gamma^\mu\ell''_L)}{\Lambda^2}, \tag{2.15}$$

where here and in what follows the superscript of the coupling denotes the combinations of chiralities involved and the subscript denotes the relevant Dirac structure. Likewise, the operator O_3 corresponds to a purely right-handed interaction

$$H_{\text{eff}}^{(RR)(RR)} = g_V^{(RR)(RR)} \frac{(\bar{\ell}_R\gamma_\mu\tau_R)(\bar{\ell}'_R\gamma^\mu\ell''_R)}{\Lambda^2}, \tag{2.16}$$

while we get a mixed term from the operator O_4

$$H_{\text{eff}}^{(LL)(RR)} = g_V^{(LL)(RR)} \frac{(\bar{\ell}_L\gamma_\mu\tau_L)(\bar{\ell}'_R\gamma^\mu\ell''_R)}{\Lambda^2} + g_V^{(RR)(LL)} \frac{(\bar{\ell}_R\gamma_\mu\tau_R)(\bar{\ell}'_L\gamma^\mu\ell''_L)}{\Lambda^2}. \tag{2.17}$$

Notice that the chirality structure $(LR)(RL)$ is not independent, since it can be Fierz rearranged into $H_{\text{eff}}^{(LL)(RR)}$.

The dim=6 radiative operators contain charged as well as neutral currents. Here we are only interested in the neutral-current component, coupling to a charged lepton pair. Switching to the physical photon and Z_0 fields and integrating out the Higgs and the Z_0, we find that we obtain a radiative operator with a photon, as well as a four-fermion contribution from Z_0 exchange. The latter is proportional to

$$\frac{v}{\Lambda^2}\frac{1}{v^2}(\bar{\ell}\sigma_{\mu\nu}\tau)q^\nu(\bar{\ell}'\gamma^\mu(g_V + g_A\gamma_5)\ell'')\,,$$

where $(g_A)\,g_V$ are the (axial)vector couplings of the Z_0 to the leptons. Taking into account the fact that $|q^\mu|$ is of the order of the τ mass, we find that this operator is suppressed relative to the leading ones by the small Yukawa coupling of the τ lepton. Thus, only the photonic contribution has to be taken into account. For this we obtain at the scale m_τ

$$H_{\text{eff}}^{\text{rad}} = \frac{e}{4\pi}\frac{v}{\Lambda^2}\sum_{h,s}g_{\text{rad}}^{(s,h)}\left(\bar{\ell}_h(-i\sigma_{\mu\nu})\tau_s\right)F^{\mu\nu}\,, \tag{2.18}$$

where $g_{\text{rad}}^{(L,R)}$ and $g_{\text{rad}}^{(R,L)}$ denote the two possible chirality combinations.[2] The matrix element for $\tau \to \ell\bar{\ell}'\ell'$ becomes

$$\langle\ell\bar{\ell}'\ell'|H_{\text{eff}}^{\text{rad}}|\tau\rangle = \alpha_{em}\frac{v}{\Lambda^2}\frac{q^\nu}{q^2}\sum_{h,s}g_{\text{rad}}^{(s,h)}\langle\ell\bar{\ell}'\ell'|\left(\bar{\ell}_h(-i\sigma_{\mu\nu})\tau_s\right)\left(\bar{\ell}'\gamma^\mu\ell'\right)|\tau\rangle\,, \tag{2.19}$$

where q is the momentum transfer through the photon. This momentum transfer is proportional to the lepton masses, and thus this contribution scales as $1/(y\Lambda^2)$ where y is a Yukawa coupling of the leptons, which would lead to an enhancement unless an additional Yukawa coupling appears in the numerator as e.g. in minimal flavour violation (see below).

Finally, we turn to the four-fermion operators with the chirality structure $(RL)(RL)$ or $(LR)(LR)$. At tree-level, they receive contributions from the dim=8 operators $P_{1,2}$, $Q_{1,2}$ and $P_{1,2}^{(T)}$, only. Therefore, their matching coefficients are further suppressed by v^2/Λ^2. The one-loop matching coefficients at the electroweak scale may also receive contributions from the dim=6 operators O_{1-4}, $R_{1,2}$, but in this case the required chirality flips induce an additional suppression by m_ℓ^2/v^2. Ignoring contributions suppressed by v^2/Λ^2 or m_ℓ^2/v^2, this reduces the number of possible Dirac structures already to six in the case where the radiative operator can contribute and to four in cases like $\tau^- \to \mu^-\mu^-e^+$, where the radiative contribution is absent. The corresponding couplings

$$g_V^{(LL)(LL)}, \quad g_V^{(RR)(RR)}, \quad g_V^{(LL)(RR)}, \quad g_V^{(RR)(LL)}, \quad g_{\text{rad}}^{(LR)}, \quad g_{\text{rad}}^{(RL)},$$

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[2]Here, for simplicity, we neglected possible form factor effects for decays into virtual photons from long-distance lepton or quark loops. In the most general case, the $\tau \to \ell\gamma^*$ vertex could be parametrized as

$$\frac{e}{4\pi}\frac{v}{\Lambda^2}\sum_{h,s}\bar{\ell}_h\left\{g_{\text{rad}}^{(s,h)}(q^2)\,(-i\sigma_{\mu\nu})\,q^\mu + m_\tau f_{\text{rad}}^{(s,h)}(q^2)\left(\gamma_\nu - \frac{q_\nu}{q^2}\slashed{q}\right)\right\}\tau_s\,,$$

where $g_{\text{rad}}^{(s,h)}(0) \equiv g_{\text{rad}}^{(s,h)}$ and $f_{\text{rad}}^{(s,h)}(0) = 0$, see, for instance, [19].

are matrices in lepton flavour space. There are in total six different decay modes of the τ^- to consider,

$$\tau^- \to e^- e^- e^+ \tag{2.20}$$

$$\tau^- \to \mu^- \mu^- \mu^+ \tag{2.21}$$

$$\tau^- \to e^- e^- \mu^+ \tag{2.22}$$

$$\tau^- \to \mu^- \mu^- e^+ \tag{2.23}$$

$$\tau^- \to \mu^- e^- e^+ \tag{2.24}$$

$$\tau^- \to e^- \mu^- \mu^+ \tag{2.25}$$

Notice that (2.20)–(2.23) contain two identical particles ($e^- e^-$ or $\mu^- \mu^-$) in the final state, whereas (2.24)+(2.25) do not. Moreover, only (2.20), (2.21), (2.24), (2.25) receive contributions from the radiative operators (2.18) via

$$\tau^- \to \ell^- \gamma^* \to \ell^- (\ell'^+ \ell'^-) \,.$$

2.1 Constraints from Minimal Flavour Violation

The flavour structure of the coupling constant has been investigated within the framework of Minimal Flavour Violation in the lepton sector (MLFV [18]). In a scenario with the minimal field content (2.1), the breaking of the lepton flavour symmetry $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_{E_R}$ is described by two spurion fields

$$\lambda = \frac{m_\ell}{v} = \frac{1}{v} \, \mathrm{diag}(m_e, m_\mu, m_\tau) \,, \tag{2.26}$$

$$g_\nu = \frac{\Lambda_{\mathrm{LN}}}{v^2} \, U^* \, \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_2}) \, U^\dagger \tag{2.27}$$

where $\lambda \sim (\bar{3}, 3)$ describes the SM Yukawa couplings of the charged leptons, and the matrix $g_\nu \sim (\bar{6}, 1)$ stems from a dim-5 lepton-number violating term,

$$\mathcal{L}_{\mathrm{Maj}} = \frac{1}{2\Lambda_{\mathrm{LN}}} \left(N^T g N \right) \,, \tag{2.28}$$

where

$$N = \left(T_3^{(R)} + \frac{1}{2} \right) H^\dagger L \tag{2.29}$$

has vanishing quantum numbers under the complete SM gauge group. If the scale Λ_{LN}, associated with lepton-number violation, is sufficiently large, the resulting neutrino masses $m_{\mathrm{Maj}} \sim v^2 / \Lambda_{\mathrm{LN}}$ are small, even if the spurion g_ν has generic entries of order unity.

We are interested in 4-lepton processes, induced by operators with some flavour structure

$$L^i L^j L_k^* L_l^* \,, \quad L^i R^j L_k^* R_l^* \,, \quad \text{etc.}$$

To render these operators formally invariant under the flavour group, they have to be multiplied by appropriate factors of λ_e and g_ν. In the following we will consider the minimal number of spurion insertions, only.[3]

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[3]This is justified as long as the spurion fields are characterized by some small expansion parameter [18, 20, 21], e.g. if the neutrino mass *differences* Δm_ν^2 are smaller than their average $\Delta \bar{m}_\nu^2$. Notice that, unlike in the case of the quark CKM matrix, the off-diagonal entries of the PMNS matrix are not always small.

Starting with the case of $L^i L^j L_k^* L_l^*$, we need at least two spurion insertions. The possible flavour structures can be read off the reduction of the $SU(3)_L$ tensor product for g_ν and $g_\nu^\dagger$,

$$\bar{6} \times 6 = 1 + 8 + 27.$$

Here the flavour singlet term corresponds to the trace of $g_\nu^\dagger g_\nu$,

$$\mathrm{tr}[g_\nu^\dagger g_\nu] = \frac{\Lambda_{\mathrm{LN}}^2}{v^4} \left(m_{\nu_1}^2 + m_{\nu_3}^2 + m_{\nu_3}^2\right) \equiv 3 \frac{\Lambda_{\mathrm{LN}}^2 \bar{m}_\nu^2}{v^4},$$

which does not induce flavour transitions at all. The flavour octet term is obtained as[4]

$$\Delta = \Delta^\dagger = g_\nu^\dagger g_\nu - \frac{1}{3} \mathrm{tr}[g_\nu^\dagger g_\nu] = \frac{\Lambda_{\mathrm{LN}}^2}{v^4} U \Delta m_\nu^2 U^\dagger. \qquad (2.30)$$

Here $\Delta m_\nu^2 = \mathrm{diag}[m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2] - \bar{m}_\nu^2$. In particular, one finds that

$$\Delta_\tau^\mu = \mathcal{O}\left(\frac{\Lambda_{LN}^2}{v^4} \Delta m_{\mathrm{atm}}^2\right)$$

whereas Δ_μ^e and Δ_τ^e are further suppressed by the neutrino mixing angle θ_{13}. It is to be stressed that Δ does neither depend on the absolute neutrino mass scale $\bar{m}_\nu^2$, nor on potential Majorana phases $\alpha_{1,2}$ in the PMNS matrix.

The flavour structure of the corresponding invariant 4-lepton operator reads $(L^* \Delta L)(L^* L)$. The coefficients of specific flavour transitions are thus given by the quadratic neutrino mass differences and PMNS elements. By the same argument, the operator $(L^* \Delta L)(R^* R)$ is invariant under the flavour group. It has been shown in [18] that Δ also drives all possible flavour transitions induced by operators that are bilinear in the lepton fields. In particular, the flavour structure of the radiative operators in (2.7), (2.8) reads $(L^* \Delta \lambda^\dagger R)$ and $(R^* \lambda \Delta L)$. Notice that the presence of a single right-handed field requires the insertion of the Yukawa spurion λ, which leads to an additional suppression factor m_ℓ/v.

Turning to the 27plet combination of g_ν and $g_\nu^\dagger$, we introduce the according representation in terms of a trace-less tensor

$$\begin{aligned}
G_{ij}^{kl} = {} & (g_\nu)_{ij} (g_\nu^*)^{kl} - \frac{1}{12} \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k\right) \mathrm{tr}(g^\dagger g) \\
& - \frac{1}{5} \left(\delta_i^a \delta_b^l \delta_j^k + \delta_j^a \delta_b^l \delta_i^k + \delta_i^a \delta_b^k \delta_j^l + \delta_j^a \delta_b^k \delta_i^l\right) \Delta_a^b
\end{aligned} \qquad (2.31)$$

with $G_{ij}^{kl} = G_{ji}^{kl} = G_{ij}^{lk}$, and $\sum_i G_{ij}^{il} = 0$. The flavour structure of the corresponding invariant 4-lepton operator reads

$$G_{ij}^{kl} L^i L^j L_k^* L_l^*.$$

In contrast to Δ, the off-diagonal matrix elements of G depend on the absolute neutrino mass scale $\bar{m}_\nu^2$ and the Majorana phases. As a consequence, in the general case, i.e. if

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[4]Notice that our definition of Δ differs from the one in [18], but only for the diagonal elements, which are irrelevant for flavour transitions.

the radiative operators do not dominate the $\tau \to 3\ell$ decay amplitudes, the purely leptonic decay modes are not directly correlated with the radiative ones $\tau \to \ell\gamma$. Relatively simple expressions for G_{ij}^{kl} can be obtained in the limit of vanishing Majorana phases, where we also employ the approximations $\sin^2\theta_{13} \sim \Delta m_{\mathrm{sol}}^2/\Delta m_{\mathrm{atm}}^2 \ll 1$ and $\theta_{23} = 45°$. For the normal neutrino hierarchy ($m_{\nu_1} \sim m_{\nu_2} \ll m_{\nu_3}$), we obtain the leading coefficients as

$$G_{\tau e}^{e\mu} \simeq -2G_{\tau\mu}^{\mu\mu} \simeq -\frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{\Delta m_{\mathrm{atm}}^2}{10}, \tag{2.32}$$

and sub-leading effects from

$$G_{\tau e}^{ee} \simeq -\frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{\sqrt{2}}{5} e^{i\delta} \sin\theta_{13} \Delta m_{\mathrm{atm}}^2, \qquad G_{\tau\mu}^{ee} \simeq \frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{\sqrt{\Delta m_{\mathrm{atm}}^2}}{2} \left(m_{\nu_{1,2}} - \cos 2\theta_{12} \frac{\Delta m_{\mathrm{sol}}^2}{2m_{\nu_{1,2}}} \right),$$

$$G_{\tau e}^{\mu\mu} \simeq \frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{\sqrt{\Delta m_{\mathrm{atm}}^2}}{2\sqrt{2}} \left(e^{i\delta} \sin\theta_{13} \sqrt{\Delta m_{\mathrm{atm}}^2} - \sin 2\theta_{12} \frac{\Delta m_{\mathrm{sol}}^2}{4m_{\nu_{1,2}}} \right),$$

$$G_{\tau\mu}^{e\mu} \simeq \frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{\sqrt{\Delta m_{\mathrm{atm}}^2}}{2} \left(\frac{3\cos\delta - 7i\sin\delta}{5} \sin\theta_{13} \sqrt{\Delta m_{\mathrm{atm}}^2} + \sin 2\theta_{12} \frac{\Delta m_{\mathrm{sol}}^2}{4m_{\nu_{1,2}}} \right). \tag{2.33}$$

where we have used the PDG parameterization [1] of the CKM matrix. For the inverted hierarchy ($m_{\nu_1} \sim m_{\nu_2} \gg m_{\nu_3}$), one has

$$G_{\tau\mu}^{ee} \simeq -5G_{\tau e}^{e\mu} \simeq 10G_{\tau\mu}^{\mu\mu} \simeq -\frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{\Delta m_{\mathrm{atm}}^2}{2}, \tag{2.34}$$

and

$$G_{\tau e}^{ee} \simeq -\frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{\Delta m_{\mathrm{atm}}^2}{\sqrt{2}} \sin\theta_{13} \frac{3\cos\delta - 7i\sin\delta}{5},$$

$$G_{\tau e}^{\mu\mu} \simeq -\frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{\Delta m_{\mathrm{atm}}^2}{2\sqrt{2}} e^{i\delta} \sin\theta_{13}, \qquad G_{\tau\mu}^{e\mu} \simeq \frac{\Lambda_{\mathrm{LN}}^2}{v^4} \frac{7\Delta m_{\mathrm{atm}}^2}{10\sqrt{2}} e^{i\delta} \sin\theta_{13}. \tag{2.35}$$

We finally note that purely right-handed lepton-flavour violating decays require at least four spurion insertions, $(R^*R)(R^*\lambda g^\dagger g \lambda^\dagger R)$, and are thus strongly suppressed in MLFV.

In summary, to obtain the dominating flavour coefficients for the operators in (2.15), (2.17), (2.18), relevant for flavour-violating τ decays in MLFV, one has to consider

$$g_V^{(L_k L^i)(L_l L^j)} \to 2c_1 \Delta_i^k \delta_j^l + c_2 G_{ij}^{kl}, \tag{2.36}$$

$$g_V^{(L_k L^i)(R_l R^j)} \to c_3 \Delta_i^k \delta_j^l, \tag{2.37}$$

$$g_{\mathrm{rad}}^{(L_k R^i)} \to c_4 \Delta_i^k, \tag{2.38}$$

whereas the chiral structures corresponding to $g_V^{(RR)(RR)}$, $g_V^{(RR)(LL)}$ and $g_{\mathrm{rad}}^{(RL)}$ are suppressed by small lepton masses. The spurion combination G_{ij}^{kl} represents a new source of LFV compared to the radiative transitions $\tau \to \ell\gamma$. While the latter are driven by the spurion Δ and hence by the difference of the squared neutrino masses, the flavour coefficients of purely left-handed four-lepton operators in MLFV also involve the absolute neutrino mass scale as well as the Majorana phases. In particular, the decay modes (2.22)+(2.23) only receive contributions from $G_{\tau\mu}^{ee}$ and $G_{\tau e}^{\mu\mu}$, respectively.
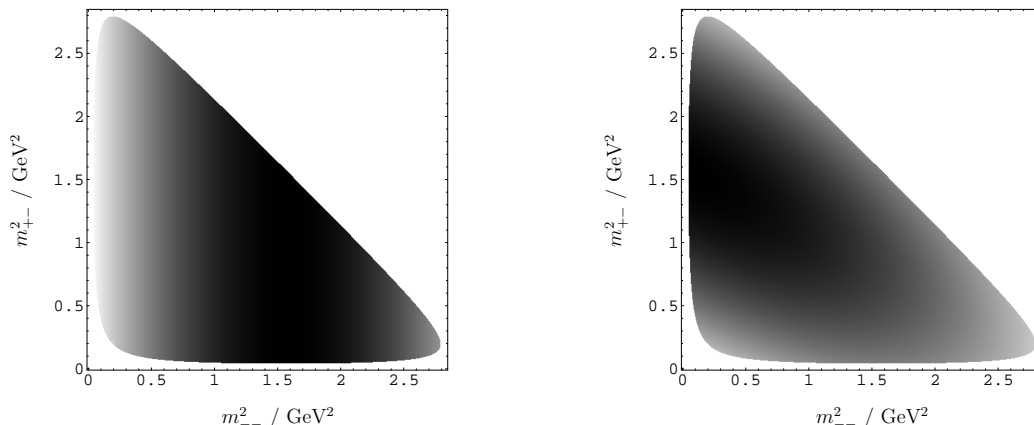
Figure 1: Dalitz plot for $\mathrm{d}^2\Gamma_V^{(LL)(LL)}$ (left) and $\mathrm{d}^2\Gamma_V^{(LL)(RR)}$ (right) in $\tau^- \to \mu^-\mu^-\mu^+$.

3. Dalitz-plot analysis

3.1 The decay $\tau^- \to \mu^-\mu^-\mu^+$

In this section we will give a detailed analysis of $\tau^- \to \mu^-\mu^-\mu^+$- as the probably most prominent channel to be looked for at the LHC. To this end, we shall consider the Dalitz distributions for the different chirality structures (2.15), (2.16), (2.17), (2.18) in the dim=6 effective Hamiltonian, in terms of the variables

$$m_{--}^2 \equiv m_{12}^2 = (p_{\mu^-} + p'_{\mu^-})^2\,, \qquad m_{+-}^2 \equiv m_{23}^2 = (p'_{\mu^-} + p_{\mu^+})^2\,, \qquad (3.1)$$

and $m_{13}^2 = m_\tau^2 + 3m_\mu^2 - m_{--}^2 - m_{+-}^2$. We will make use of (approximate) helicity conservation, which implies that many of the interference terms between the operators with different chiralities are suppressed by powers of m_μ, and can be ignored to first approximation.

In the simplest case all four leptons are left-handed, and the decay amplitude is determined by $H_{\mathrm{eff}}^{(LL)(LL)}$ in (2.15). The corresponding Dalitz distribution

$$\frac{\mathrm{d}^2\Gamma_V^{(LL)(LL)}}{\mathrm{d}m_{23}^2\,\mathrm{d}m_{12}^2} = \frac{|g_V^{(L_\mu L^\tau)(L_\mu L^\mu)}|^2}{\Lambda^4}\,\frac{(m_\tau^2 - m_\mu^2)^2 - (2m_{12}^2 - m_\tau^2 - 3m_\mu^2)^2}{256\,\pi^3\,m_\tau^3}\,, \qquad (3.2)$$

is shown in figure 1 (left). The events are equally distributed along m_{+-}^2, while there is a rather flat maximum at $m_{--}^2 = m_{12}^2 \simeq m_\tau^2/2$. The case with all particles right-handed is completely analogous and yields the same distribution with $g_V^{(LL)(LL)} \to g_V^{(RR)(RR)}$. (We remind the reader that $g_V^{(RR)(RR)}$ is expected to be strongly suppressed within MLFV scenarios.)

Next we will consider the operator $H_{\mathrm{eff}}^{(LL)(RR)}$ in (2.17). For a left-handed τ-lepton we obtain the Dalitz distribution

$$\frac{\mathrm{d}^2\Gamma_V^{(LL)(RR)}}{\mathrm{d}m_{23}^2\mathrm{d}m_{12}^2} = \frac{|g_V^{(L_\mu L^\tau)(R_\mu R^\mu)}|^2}{\Lambda^4}\left[\frac{(m_\tau^2 - m_\mu^2)^2 - 4m_\mu^2\,(m_\tau^2 + m_\mu^2 - m_{12}^2)}{512\,\pi^3\,m_\tau^3}\right.$$
$$\left. -\frac{(2m_{13}^2 - m_\tau^2 - 3m_\mu^2)^2 + (2m_{23}^2 - m_\tau^2 - 3m_\mu^2)^2}{1024\,\pi^3\,m_\tau^3}\right]\,, \qquad (3.3)$$
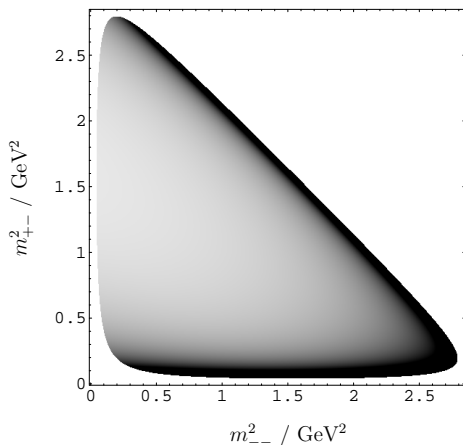
Figure 2: Dalitz plot for $\mathrm{d}^2\Gamma_{\rm rad}^{(LR)}$ in $\tau^- \to \mu^-\mu^-\mu^+$.

shown in figure 1 (right). In this case the events are distributed around a flat maximum at $m_{+-}^2 \simeq m_\tau^2/2$ and $m_{--}^2 \simeq 0$. Again, the case of a right-handed τ yields the same distribution. As pointed out above, the interference terms between (2.15) and (2.17) are suppressed by m_μ^2/m_τ^2.

In addition to the four-fermion operators we also get a contribution from the radiative interaction via (2.19). The resulting Dalitz distribution for a right-handed τ-lepton,

$$
\frac{\mathrm{d}^2\Gamma_{\rm rad}^{(LR)}}{\mathrm{d}m_{23}^2\mathrm{d}m_{12}^2} = \alpha_{\rm em}^2 \frac{|g_{\rm rad}^{(L_\mu R^\tau)}|^2 v^2}{\Lambda^4} \left[\frac{m_\mu^2(m_\tau^2-m_\mu^2)^2}{128\pi^3 m_\tau^3}\left(\frac{1}{m_{13}^4} + \frac{1}{m_{23}^4} \right) + \frac{m_\mu^2(m_\tau^4-3m_\tau^2 m_\mu^2 + 2m_\mu^4)}{128\pi^3 m_{13}^2 m_{23}^2 m_\tau^3} \right.
$$
$$
\left. + \frac{(m_{13}^2 + m_{23}^2)(m_{12}^4 + m_{13}^4 + m_{23}^4 - 6m_\mu^2(m_\mu^2 + m_\tau^2))}{256\pi^3 m_{13}^2 m_{23}^2 m_\tau^3} + \frac{2m_{12}^2 - 3m_\mu^2}{128\pi^3 m_\tau^3} \right],
$$

(3.4)

is plotted in figure 2. Due to the photon pole, the events are concentrated at low values of m_{23}^2 or m_{13}^2, respectively. Again the decay of the left-handed τ is completely analogous.

Finally, we have to take into account the contributions from the interference terms between the radiative operators and four-fermion operators, for the cases where only the chirality of the τ-lepton has to be flipped. The interference term between (2.15) and (2.18) reads

$$
\frac{\mathrm{d}^2\Gamma_{\rm mix}^{(LL)(LL)}}{\mathrm{d}m_{23}^2\mathrm{d}m_{12}^2} = \alpha_{\rm em} \frac{2v\,\mathrm{Re}[g_{\rm V}^{(L_\mu L^\tau)(L_\mu L^\mu)} g_{\rm rad}^{*(L_\mu R^\tau)}]}{\Lambda^4} \left[\frac{m_{12}^2 - 3m_\mu^2}{64\pi^3 m_\tau^2} + \frac{m_\mu^2(m_\tau^2 - m_\mu^2)(m_{13}^2 + m_{23}^2)}{128\pi^3 m_\tau^2 m_{13}^2 m_{23}^2} \right],
$$

(3.5)

The interference between (2.17) and (2.18) results in

$$
\frac{\mathrm{d}^2\Gamma_{\rm mix}^{(LL)(RR)}}{\mathrm{d}m_{23}^2\mathrm{d}m_{12}^2} = \alpha_{\rm em} \frac{2\,v\,\mathrm{Re}[g_{\rm V}^{(L_\mu L^\tau)(R_\mu R^\mu)} g_{\rm rad}^{*(L_\mu R^\tau)}]}{\Lambda^4}
$$
$$
\times \left[\frac{m_\tau^2 - m_{12}^2 - 3m_\mu^2}{256\,\pi^3\,m_\tau^2} + \frac{m_\mu^2(m_\tau^2 - m_\mu^2)(m_{13}^2 + m_{23}^2)}{256\,\pi^3\,m_\tau^2\,m_{13}^2\,m_{23}^2} \right].
$$

(3.6)

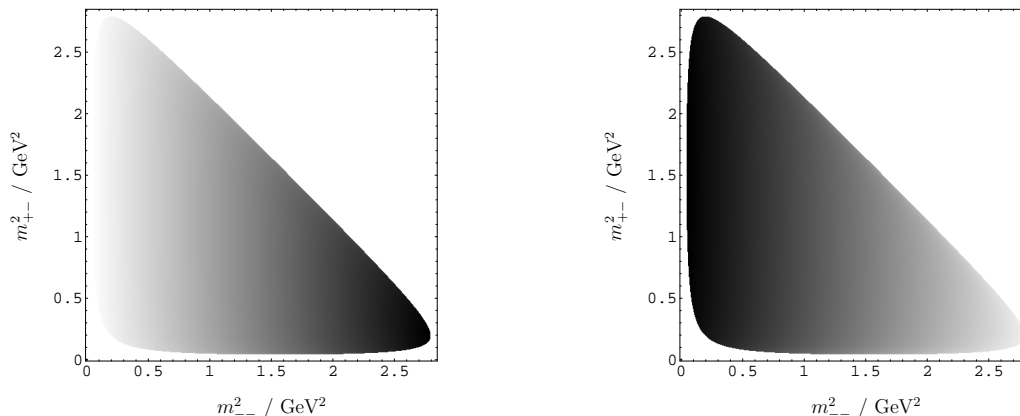**Figure 3:** Dalitz plot for $|\mathrm{d}^2\Gamma_{\mathrm{mix}}^{(LL)(LL)}|$ (left) and $|\mathrm{d}^2\Gamma_{\mathrm{mix}}^{(LL)(RR)}|$ (right) in $\tau^- \to \mu^-\mu^-\mu^+$.

In both cases, the photon pole at $m_{13}^2 = 0$ or $m_{23}^2 = 0$ is suppressed by the small muon mass. The remaining terms increase (decrease) monotonically with m_{12}^2, respectively, see figure 3.

Combining (3.2)–(3.6) and integrating over phase space, we obtain for the total decay width (normalized to the SM decay $\tau \to \mu\bar{\nu}_\mu\nu_\tau$, and neglecting the muon mass)

$$
\frac{\Gamma[\tau^- \to \mu^-\mu^-\mu^+]}{\Gamma[\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau]} = \frac{1}{G_F^2 \Lambda^4} \left\{ \frac{2\,|g_V^{(LL)(LL)}|^2 + 2\,|g_V^{(RR)(RR)}|^2 + |g_V^{(LL)(RR)}|^2 + |g_V^{(RR)(LL)}|^2}{8} \right.
$$
$$
+ \frac{\alpha_{\mathrm{em}}^2 v^2}{m_\tau^2} \left(\ln \frac{m_\tau^2}{m_\mu^2} - \frac{11}{4} \right) \left(|g_{\mathrm{rad}}^{(LR)}|^2 + |g_{\mathrm{rad}}^{(RL)}|^2 \right) \tag{3.7}
$$
$$
\left. + \frac{\alpha_{\mathrm{em}}\,v}{2m_\tau} \mathrm{Re} \left[2\,g_{\mathrm{rad}}^{*(LR)}\,g_V^{(LL)(LL)} + g_{\mathrm{rad}}^{*(LR)}\,g_V^{(LL)(RR)} + (L \leftrightarrow R) \right] \right\}.
$$

This result is consistent with the formula quoted, for instance, in [10].

3.2 The decay $\tau^- \to e^-\mu^-\mu^+$

For completeness, we also discuss the Dalitz distributions for the decay mode $\tau^- \to e^-\mu^-\mu^+$, which belongs to the class of decays with three *different* particles in the final state. Again, we will give the results in terms of the invariant masses

$$
m_{--}^2 \equiv m_{12}^2 = (p_{e^-} + p'_{\mu^-})^2\,, \qquad m_{+-}^2 \equiv m_{23}^2 = (p'_{\mu^-} + p_{\mu^+})^2\,, \tag{3.8}
$$

and $m_{13}^2 = m_\tau^2 + 2m_\mu^2 - m_{--}^2 - m_{+-}^2$, where we set the electron mass to zero.[5]

From the purely left-handed term in the effective Hamiltonian, $H_{\mathrm{eff}}^{(LL)(LL)}$ in (2.15), we obtain the Dalitz distribution

$$
\frac{\mathrm{d}^2\Gamma_V^{(LL)(LL)}}{\mathrm{d}m_{23}^2\,\mathrm{d}m_{12}^2} = \frac{|g_V^{(L_e L^\tau)(L_\mu L^\mu)}|^2}{\Lambda^4} \frac{m_\tau^4 - (2m_{12}^2 - m_\tau^2 - 2m_\mu^2)^2}{512\,\pi^3\,m_\tau^3}\,, \tag{3.9}
$$

which (except in the vicinity of the phase-space boundaries) coincides with (3.2) up to corrections of order m_μ^2/m_τ^2 and a statistical factor. Consequently, the corresponding Dalitz plot looks almost identical to figure 1 (left).

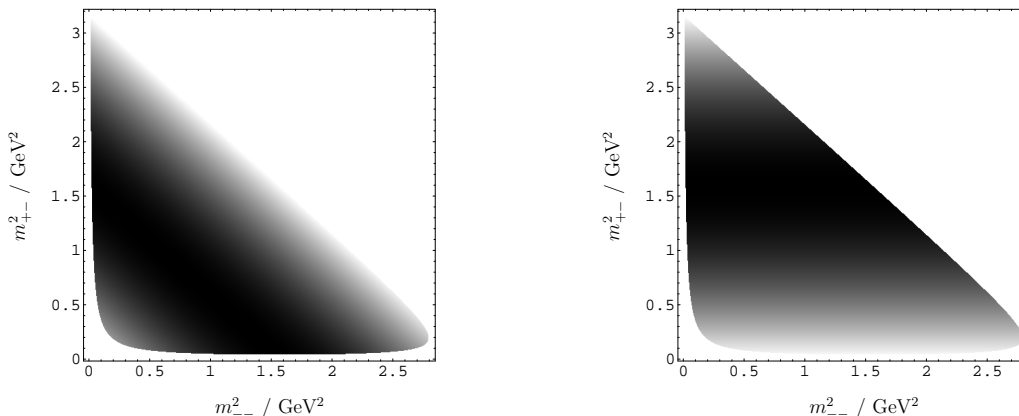[5]Notice that the photon pole from $\tau^- \to e^-\gamma^* \to e^-\mu^-\mu^+$ is still regulated by the muon mass.

Figure 4: Dalitz plots for the two contributions to $\mathrm{d}^2\Gamma_V^{(LL)(RR)}$ in $\tau^- \to e^-\mu^-\mu^+$.

From $H_{\mathrm{eff}}^{(LL)(RR)}$ in (2.17) we obtain for the case of a left-handed τ-lepton

$$
\frac{\mathrm{d}^2\Gamma_V^{(LL)(RR)}}{\mathrm{d}m_{23}^2\mathrm{d}m_{12}^2} = \frac{|g_V^{(L_eL^\tau)(R_\mu R^\mu)}|^2}{\Lambda^4} \frac{m_\tau^4 - (2m_{13}^2 - m_\tau^2 - 2m_\mu^2)^2}{512\,\pi^3\,m_\tau^3}
$$
$$
+ \frac{|g_V^{(L_\mu L^\tau)(R_e R^\mu)}|^2}{\Lambda^4} \frac{(m_\tau^2 - 2m_\mu^2)^2 - (2m_{23}^2 - m_\tau^2 - 2m_\mu^2)^2}{512\,\pi^3\,m_\tau^3}\,. \tag{3.10}
$$

The corresponding Dalitz plots for the two separate contributions are shown in figure 4 (left: the term $\propto |g_V^{(L_eL^\tau)(R_\mu R^\mu)}|^2$; right: the term $\propto |g_V^{(L_\mu L^\tau)(R_e R^\mu)}|^2$). The events are distributed around $m_{--}^2 + m_{+-}^2 \simeq m_\tau^2/2$ or $m_{+-}^2 \simeq m_\tau^2/2$, respectively. For equal coupling constants in (3.10) we recover the $\tau \to 3\mu$ case in (3.3) (again up to mass corrections and a statistical factor).

For the radiative decay operators, we obtain

$$
\frac{\mathrm{d}^2\Gamma_{\mathrm{rad}}^{(LR)}}{\mathrm{d}m_{23}^2\mathrm{d}m_{12}^2} = \alpha_{\mathrm{em}}^2 \frac{|g_{\mathrm{rad}}^{(L_eR^\tau)}|^2\,v^2}{\Lambda^4} \left[\frac{m_\mu^2\,(m_{23}^2 - m_\tau^2)^2}{64\,\pi^3\,m_\tau^3\,m_{23}^4} + \frac{m_{12}^4 + m_{13}^4 - 2m_\mu^4}{128\,\pi^3\,m_\tau^3\,m_{23}^2} + \frac{m_\tau^2 - m_{23}^2}{128\,\pi^3\,m_\tau^3} \right], \tag{3.11}
$$

and the corresponding Dalitz plot is shown in Fig: 5. In this case the photon pole enhances the events at low values of $m_{+-}^2 = m_{23}^2$.

Finally, for the interference terms between (2.15) and (2.18) we get

$$
\frac{\mathrm{d}^2\Gamma_{\mathrm{mix}}^{(LL)(LL)}}{\mathrm{d}m_{23}^2\mathrm{d}m_{12}^2} = \alpha_{\mathrm{em}} \frac{2\,v\,\mathrm{Re}[g_V^{(L_eL^\tau)(L_\mu L^\mu)}\,g_{\mathrm{rad}}^{*(L_eR^\tau)}]}{\Lambda^4} \left[\frac{m_{12}^2 - 2m_\mu^2}{128\pi^3 m_\tau^2} + \frac{m_\mu^2}{128\pi^3 m_{23}^2} \right], \tag{3.12}
$$

and

$$
\frac{\mathrm{d}^2\Gamma_{\mathrm{mix}}^{(LL)(RR)}}{\mathrm{d}m_{23}^2\mathrm{d}m_{12}^2} = \alpha_{\mathrm{em}} \frac{2\,v\,\mathrm{Re}[g_V^{(L_eL^\tau)(R_\mu R^\mu)}\,g_{\mathrm{rad}}^{*(L_eR^\tau)}]}{\Lambda^4} \left[\frac{m_{13}^2 - 2m_\mu^2}{128\pi^3 m_\tau^2} + \frac{m_\mu^2}{128\pi^3 m_{23}^2} \right]. \tag{3.13}
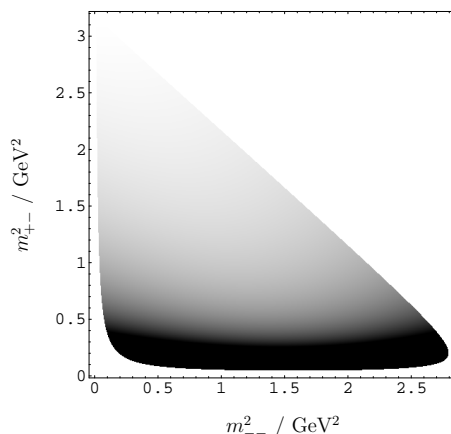$$

The corresponding Dalitz plots are shown in figure 6.

Figure 5: Dalitz plot for $\mathrm{d}^2\Gamma_{\mathrm{rad}}^{(LR)}$ in $\tau^- \to e^- \mu^- \mu^+$.
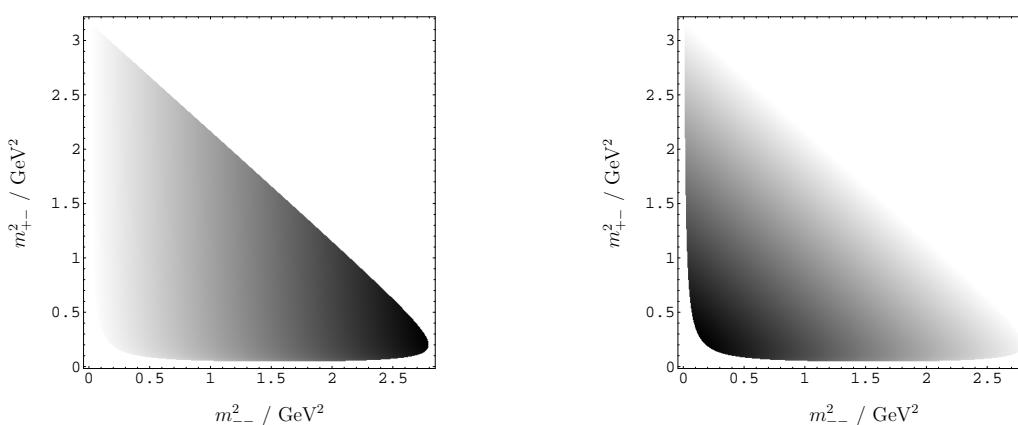

Figure 6: Dalitz plot for $|\mathrm{d}^2\Gamma_{\mathrm{mix}}^{(LL)(LL)}|$ (left) and $|\mathrm{d}^2\Gamma_{\mathrm{mix}}^{(LL)(RR)}|$ (right) in $\tau^- \to e^- \mu^- \mu^+$.

4. Discussion and conclusions

Lepton-number violating processes, like the decay $\tau^- \to \ell\ell'\ell''$ discussed in this paper, provide an important test of the Standard Model. Since many new physics models allow for dramatic enhancements compared to the tiny SM effects, there is a potential to falsify the SM by measuring such decays at future experiments. At the same time, the foreseen improvement of experimental limits may further tighten the constraints on specific new physics models.

In both cases, the event distributions in phase space provided in this work, will be helpful. The difference between the Dalitz distributions arising from four-lepton operators, which show a rather uniform behaviour, and the distributions from radiative operators which are concentrated at small values of $m_{+-}^2 = (p_{\ell^+} + p_{\ell^-})^2$, clearly gives a handle to disentangle different new physics models already on the basis of rather few events. Typically, these models give rather different predictions for the relative size of radiative and four-fermion operators, as we will discuss in the following.

In super-symmetric extensions of the SM, one typically finds that the photon-dipole

operator, induced by penguin diagrams, dominates over the four-lepton operators. This leads to simple correlations like (see e.g. [10])

$$\frac{\Gamma(\tau \to 3\mu)}{\Gamma(\tau \to \mu\gamma)} \simeq \frac{\alpha_{\text{em}}}{3\pi}\left(\ln\frac{m_\tau^2}{m_\mu^2} - \frac{11}{4}\right) = \mathcal{O}(10^{-3}).$$

In this case, one expects Dalitz distributions as shown in figure 2.

It has been pointed out in [13] that Higgs-mediated $\tau \to \mu$ transitions may alter this result, if $\tan\beta$ and the off-diagonal slepton mass-matrix element $\delta_{3\ell}$ are large. For instance, in the decoupling limit ($\cos(\beta - \alpha) = 0$, $m_{A^0} \gg M_Z$) the author of [13] finds

$$\frac{\Gamma(\tau \to \ell\mu\mu)}{\Gamma(\tau \to \ell\gamma)} \leq \frac{3 + 5\delta_{\ell\mu}}{36} \sim \mathcal{O}(0.1)$$

where $\delta_{\ell\mu} = \tilde{m}_{\ell\mu}^2/\tilde{m}^2$. Testing such scenarios in experiment will be more involved, as one generally has to allow for the interplay of all contributions to the Dalitz distributions, figure 1–3.

The situation is somewhat different in the case of Little Higgs Models with T-Parity (LHT) [15 – 17]. Here the Z_0 and box-diagram contributions dominate compared to the radiative operators [17], which is mainly due to the constructive (respectively destructive) interference between the individual heavy gauge boson contributions. Depending on the parameter values of the LHT, one finds

$$\frac{\Gamma(\tau \to 3\mu)}{\Gamma(\tau \to \mu\gamma)} = \mathcal{O}(1)$$

for a mass scale of the LHT mirror fermions of about $1\,\text{TeV}$. In this case, one can expect rates for LFV decays which are already close to the present bounds. Because of the sub-dominance of the radiative dipole operator, we expect a rather flat Dalitz distribution for $\tau \to 3\mu$, as illustrated in figure 1.

Acknowledgments

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